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Research Article

Preservice Mathematics Teachers' Context-Based Problems and Representations for Common Multiples and Common Divisors

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Abstract

This study adopted a qualitative approach to explore how pre-service mathematics teachers make sense of the concepts of common multiples and common divisors through context-based problems and the mathematical representations they use during problem solving. The research was conducted as a case study. The participants consisted of 58 third-year pre-service mathematics teachers enrolled at a public university during the fall semester of the 2025–2026 academic year. Data were collected by asking the participants to create real-life problem situations and solve the problems they had created using appropriate mathematical representations. The data were analyzed through content analysis. The findings revealed that most pre-service mathematics teachers were able to generate problems involving common multiples and common divisors. However, some participants showed a tendency to formulate problems related to the concepts of least common multiple and greatest common divisor. Time- and measurement-related situations emerged as the most frequently preferred contexts. In terms of mathematical representations, participants most frequently used tables, number lines, the factorization algorithm, the rainbow model, and fraction strips, although the majority relied solely on procedural solutions. Overall, the findings emphasize the importance of supporting conceptual understanding through meaningful contexts and the use of multiple mathematical representations in the teaching of common multiples and common divisors.

Keywords: Common divisor, common multiple, context-based problems, preservice teachers, types of representation.

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INTRODUCTION

In mathematics teaching, it is essential that concepts are not only considered at an operational level, but also within a context, to develop students' higher-order thinking skills (Hiebert& Carpenter, 1992). Context is the source of information that enables the appropriate interpretation of situations related to the subject being studied (Goffman, 1974). Context-Based Teaching has been identified as an effective instructional approach because it uses real-life contexts to support students' understanding of mathematical concepts (Glynn & Koballa, 2005). Accordingly, students should be provided with well-designed context-based problems that support meaningful conceptual understanding throughout the learning process. Benckert (1997) proposed several principles for designing context-based problems. These principles continue to be recognized in contemporary studies on mathematical problem posing and the design of meaningful problem situations (Baumanns & Rott, 2021). Accordingly, the following design principles suggested by Benckert (1997) were considered in this study: The problem should reflect a logical situation that the student is facing and needs to solve:

- The problem should contain all elements that are directly related to and meaningful in real life.
- The problem should be multi-step, so that it cannot be solved in one step.
- The problem's solution process can include more information than is necessary.
- The problem should not directly and explicitly state the unknown (the desired outcome).

Accordingly, it is necessary for learners to encounter problems that are appropriately designed to facilitate understanding throughout their learning process (Cumming & Maxwell, 1999). Hill (1998) also emphasized the importance of ensuring that the problems designed are relevant to real-life situations.

The ability to identify the appropriate mathematical structure when faced with a problem and to explain this choice along with its rationale is considered one of the fundamental indicators of mathematical understanding (Schoenfeld, 1985, 1992). However, the literature indicates that while students are often able to apply the correct procedures, they struggle to justify the appropriateness of these procedures and largely carry out their solution processes at the level of procedural knowledge (Hiebert, 2003; Lithner, 2008).

In this context, concepts based on multiplicative structures such as common divisors and common multiples play a central role in helping students understand divisibility, cyclic repetition, and equal distribution. The teaching of these concepts requires a multidimensional cognitive process that involves not only reaching the correct result but also analyzing the problem context, identifying the appropriate mathematical structure, and justifying this choice (Lithner, 2008; Schoenfeld, 1985; Stylianides, 2007). Indeed, it has been noted that students often produce solutions based on procedural knowledge regarding common multiples and common divisors; they experience various limitations in interpreting the meaning of these concepts, their conditions of use, and their contextual appropriateness (Zazkis & Campbell, 1996). Furthermore, it has been observed that although students possess foundational knowledge of the concepts of factors and multiples, they experience difficulties in applying these concepts to different real-life contexts and explaining their mathematical reasoning (Akın & İlhan, 2020; Hurst et al, 2021). Furthermore, it is emphasized that the correct understanding of a mathematical situation is possible not only through symbolic operations but also by transforming the context into a mathematical structure. In this transformation process, mathematical representations play a decisive role in supporting conceptual understanding (Ainsworth, 2006; Duval, 2006). Mathematical representations enable learners to express and interpret mathematical ideas in different forms. These representations may include tables, number lines, symbolic representations, and visual models. Using multiple representations supports learners in establishing connections between different forms of mathematical thinking and contributes to the development of conceptual understanding (Ainsworth, 2006; Duval, 2006). In this regard, how students interpret the concepts of common multiples and common divisors in everyday contexts, in which situations they apply these concepts, and how they justify their choices have emerged as an important area of research. The “interpretation” dimension emphasized in the learning outcomes of the Turkey Century Education Model [TCEM] elementary mathematics curriculum aligns with this theoretical framework, which centers on the contextual understanding and justification of mathematical concepts (Ministry of National Education [MoNE], 2024). In the TCEM, the concepts of common multiples and common divisors are included within the “Numbers and Quantities” theme at the 6th-grade level. While students are expected to interpret and use the common multiples and common divisor of two natural numbers when solving problems, it is noted that the concepts of least common multiple (LCM) and greatest common divisor (GCD) will not be addressed (MoNE, 2024). This represents a significant difference compared to the 2018 curriculum. In this context, examining students' processes of interpreting concepts and formulating problems is important for revealing the nature of mathematical understanding.

Since problem-posing and problem-solving are frequently encountered in mathematics education and daily life, there is a clear need to develop these skills. Consequently, the ability of pre-service teachers to effectively utilize these skills will play a significant role in their design of effective teaching processes. From this perspective, previous studies have primarily focused on students' achievement and conceptual understanding of common multiples and common divisors (e.g., Bağcı & Ay, 2024; Çubukluöz et al., 2018; Duman & Es, 2023; Gökçen, 2009; Keyik, 2023). However, limited attention has been given to the context-based problems created by pre-service mathematics teachers and the mathematical representations they use when solving these problems. In this study, context-based problems created by pre-service mathematics teachers regarding the concepts of common multiples and common divisors, as well as the representations they used in solving these problems, were examined. In this regard, the purpose of this study is to examine the context-based problems created by pre-service mathematics teachers regarding the concepts of common multiples and common divisors, with a particular focus on the contexts used in these problems and the representations employed in their solutions.

CONCEPTUAL FRAMEWORK

The concepts of common multiples and common divisors are directly related to divisibility, factor–multiple relationships, and multiplicative reasoning, and constitute a conceptual domain that can be addressed within the framework of Vergnaud's theory of multiplicative structures (Vergnaud, 1983). According to this theory, students' development of mathematical understanding depends not only on procedural skills but also on their ability to distinguish which mathematical structure is appropriate for different types of problems. Situations requiring a common divisor—such as equal division, determining a common unit of measure, and partitioning quantities into equal-sized groups—and situations requiring a common multiple—such as synchronization, periodic repetition, and occurring simultaneously—constitute the two fundamental structures of this conceptual domain.

Learning divisibility and multiplicative structure knowledge in a fragmented or rule-memorization-based manner makes it difficult for students to make this contextual distinction.

Zazkis & Campbell (1996) note that students often use their knowledge of factors and divisibility independently of context, and that this can lead to incorrect concept selection in terms such as common divisors and common multiples. Based on students' responses to context-based tasks, Martínez et al. (2022) found that the problem context directly influenced students' use of common divisors and that contextual cues played a decisive role in concept selection.

According to Schoenfeld (1985, 1992), selecting the appropriate mathematical concept during the problem-solving process is one of the most critical stages of problem solving. Even when students know how to perform mathematical procedures, they may fail to solve a problem successfully if they are unable to identify which mathematical concept or strategy is appropriate for the given problem situation. Lithner (2008) explains this situation by noting that students tend to prefer solution strategies based on procedural reasoning and avoid conceptual justification. In the context of common multiples and common divisors, this manifests as students being able to answer the question "how is it calculated?" but failing to justify the question "why this concept?"

According to the TCEM, the MAT.6.1.4 learning outcome is structured around interpretation as a core mathematical competence. Its process components guide students from understanding real-life contexts to interpreting mathematical relationships and expressing these relationships through appropriate representations. This structure provides an appropriate framework for examining how pre-service mathematics teachers construct context-based problems and employ mathematical representations in their solutions. Mathematical explanation and the use of representations are recognized as important indicators of students' conceptual understanding (Stylianides, 2007; Yackel & Hanna, 2003). The representations used in this process also play a critical role in meaningful learning. Duval (2006) and Ainsworth (2006) note that students' ability to transition between verbal, procedural, visual, and tabular representations facilitates a deeper understanding of mathematical concepts. When reviewing studies on common multiples and common divisors in the literature, research conducted with middle school students generally stands out (Bağcı & Ay, 2024; Çubukluöz et al., 2018; Duman & Es, 2023; Gökçen, 2009; Keyik, 2023). These studies have generally examined students' achievement levels regarding these concepts. No studies have been found that examine pre-service teachers' levels of problem-solving, interpretation, and representation regarding these concepts.

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In line with this theoretical framework, it is important to investigate pre-service teachers' abilities to create, interpret, and represent context-based problems related to the concepts of common divisors and common multiples. As future mathematics teachers, their conceptual understanding and instructional decisions are likely to influence how these concepts are taught in classrooms. Examining how pre-service teachers construct context-based problems and use mathematical representations may provide valuable insights into their conceptual understanding and help identify areas that require further support in teacher education programs. Therefore, the purpose of this study is to examine the context-based problems created by pre-service mathematics teachers regarding the concepts of common multiples and common divisors, as well as the types of representations they use in solving these problems. In line with this purpose, answers were sought to the following research questions:

1. What types of contexts do pre-service middle school mathematics teachers use when creating context-based problems related to the concepts of common multiples and common divisors?

2. What mathematical representations do pre-service middle school mathematics teachers use when solving the context-based problems they create regarding the concepts of common multiples and common divisors?

METHODOLOGY

Research Design

This study employed a qualitative single-case study design. A case study enables an in-depth investigation of a bounded system within its real-life context (Merriam, 1998; Yin, 2018). In this study, the case was defined as the context-based problem-posing practices of a cohort of third-year pre-service middle school mathematics teachers. These practices were examined within the context of the concepts of common multiples and common divisors. The case was bounded by a single cohort of pre-service middle school mathematics teachers enrolled in an Teaching of Numbers course at a state university. A single-case design was considered appropriate because it allowed for an in-depth examination of participants' context-based problem-posing practices and their use of mathematical representations within this specific educational context.

Study Group

The participants of this study consisted of 58 third-year pre-service middle school mathematics teachers enrolled at a state university in Türkiye. Third-year pre-service teachers were selected because they had completed the Teaching of Numbers course, in which they studied the concepts of common multiples and common divisors within the framework of the middle school mathematics curriculum. Accordingly, they possessed the prerequisite content knowledge required to create context-based problems and use appropriate mathematical representations related to these concepts. Data were collected during the implementation of the Teaching of Numbers course in the 2025–2026 academic year, after the participants had completed the instructional content related to the concepts of common multiples and common divisors. To ensure participant anonymity, each pre-service mathematics teacher was assigned an identification code ranging from PSMT1 to PSMT58 (Pre-Service Mathematics Teacher). These codes are used throughout the Findings section when presenting representative examples from the participants' written responses."

Data Collection Tools

Data were collected through a written data collection form consisting of open-ended questions. The form was developed in accordance with the MAT.6.1.4 learning outcome in the Turkish Century Education Model (TCEM) Mathematics Curriculum and was reviewed by two experts in mathematics education to ensure its content validity. Based on their feedback, minor revisions were made to improve the clarity and comprehensibility of the questions.

Participants were asked to create context-based problems in accordance with the MAT.6.1.4 learning outcome, which focuses on interpreting common multiples and common divisors through real-life problems or mathematical situations, and to provide detailed explanations of their solutions. The written responses constituted the primary data source of the study.

Data Analysis and Data Collection Process

The data were analyzed using qualitative content analysis. The context categories were generated inductively from the participants' written responses. Subsequently, the identified contexts were interpreted in relation to the characteristics of context-based problems proposed by Benckert (1997). Likewise, the mathematical representations used in the solutions were analyzed inductively, with the representation categories emerging from the participants' written responses.

The unit of analysis was each participant's written response, including the context-based problem, its solution, and the explanation for the selected mathematical representation. All responses were coded independently by two researchers. To ensure the reliability of the coding process, inter-coder agreement was calculated using the Miles and Huberman (1994) formula, yielding an agreement rate of 92%, which indicates a high level of reliability. Following the independent coding, the researchers discussed the remaining discrepancies and reached consensus on the final coding structure.

Frequencies were calculated only to describe the distribution of the identified categories and to support the qualitative interpretation of the findings. They were not used for statistical inference but rather to indicate the prevalence of the categories identified through the content analysis.

FINDINGS

The findings of this study consist of the context-based problems designed by pre-service teachers regarding the concepts of common multiples and common divisors, the contexts used in these problems, and the mathematical representations employed in their solutions. The findings are presented under two main headings based on the mathematical concepts examined: (1) common multiples and (2) common divisors. Under each heading, the contexts used in the problems and the mathematical representations employed in their solutions are reported in accordance with the research questions.

Findings Regarding Common Multiples

In the findings regarding the written responses provided by the pre-service teachers on the concept of common multiples, consideration was given to whether the learning outcomes were met, whether an appropriate context was provided within the problem statement, and whether the mathematical representations used in solving the problem. Table 1 first presents the frequencies and percentages, indicating the extent to which the pre-service teachers' written responses met the learning outcomes.

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Table 1.

Types and Frequency Distributions of Problems Designed by Pre-service Teachers Related to the Concept of Common Multiples

Problem Types	Frequency (f)	Percentage (%)
Common Multiple Problems	41	70,7
Least Common Multiple (LCM) Problems	7	12,1
Problems Unrelated to the Target Concept	2	3,4
No Response	8	13,8

An examination of Table 1 reveals that the problems designed by 41 pre-service teachers included the concept of common multiples. Seven pre-service teachers created problems involving the least common multiple (LCM) instead of common multiples. It was determined that 10 pre-service teachers either created problems unrelated to the concept or did not create any problems at all.

The pre-service teachers created common multiple problems using different contexts. Table 2 presents the codes related to the themes of the contexts used by the 41 t pre-service teachers in the problems that included the concept of common multiples.

Table 2.

Codes and Frequency Distributions of the Contexts Used by Pre-service Teachers in Common Multiple Problems

Codes	Frequency (f)	Percentage (%)
Time	26	63,4
Measurement Units (Length / Weight / Area)	9	22
Number	6	14,6

Responses representing different context codes were identified among the 41 pre-service teachers who were able to create context-based problems related to common multiples. As presented in Table 2, twenty-six pre-service teachers designed problems coded as Temporal Context by incorporating recurring time-related situations into their problem scenarios. Nine problems were coded under Measurement Context, including contexts involving length, weight, and area. In addition, six pre-service teachers created problems involving common multiples but without embedding them in any real-life context.

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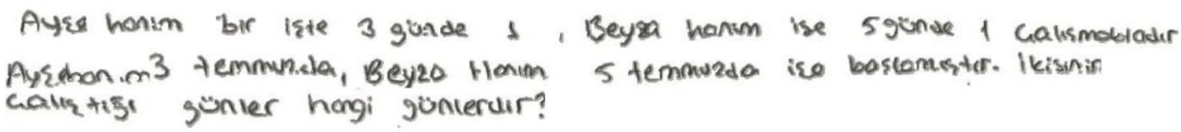
An example of the Temporal Context code is presented in Figure 1. PSMT10 designed a problem based on recurring events occurring at regular time intervals, requiring the use of the concept of common multiples within a meaningful real-life context.

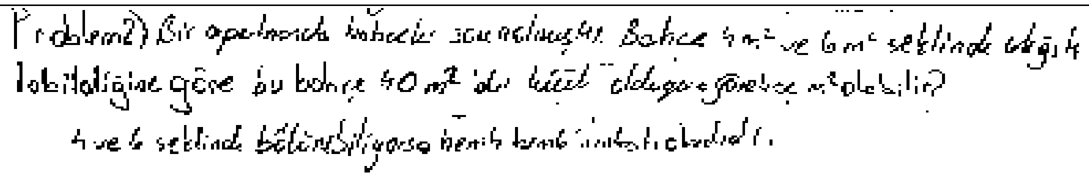


"Ayşe goes to work every 3 days, while Beyza goes to work every 5 days. Ayşe started work on 3 July, and Beyza started work on 5 July. On which dates will they both be at work?"

Figure 1. *The Common Multiple Problem Designed by PSMT10 in a Temporal Context.*

In the problem designed for PSMT10, the working days of two individuals (Ayşe Hanım and Beyza Hanım), which occur at specific intervals, are given, and the task is to determine the days on which these schedules overlap. The problem requires the use of a tool to determine the intersection days of recurring events corresponding to different day cycles. Additionally, by providing different starting days, the problem encourages students to develop various strategies, such as creating a timeline and listing consecutive days. With these features, the problem presents the concept of common multiples through a real-life context that requires identifying a common multiple to solve the given situation. Therefore, such problems may provide students with opportunities to relate the concept of a common multiple to determining the common occurrence of periodic events with different starting points.

Figure 2 shows a real-life problem in which PSMT12 relates the concept of common multiples within the context of units of measurement.



"The garden plots in an apartment complex are to be divided into sections of 4 m² and 6 m². If the area of a garden is less than 40 m², what could its possible area be?"

Figure 2. *The Common Multiple Problem Designed by PSMT12 in the Context of Measurement Units.*

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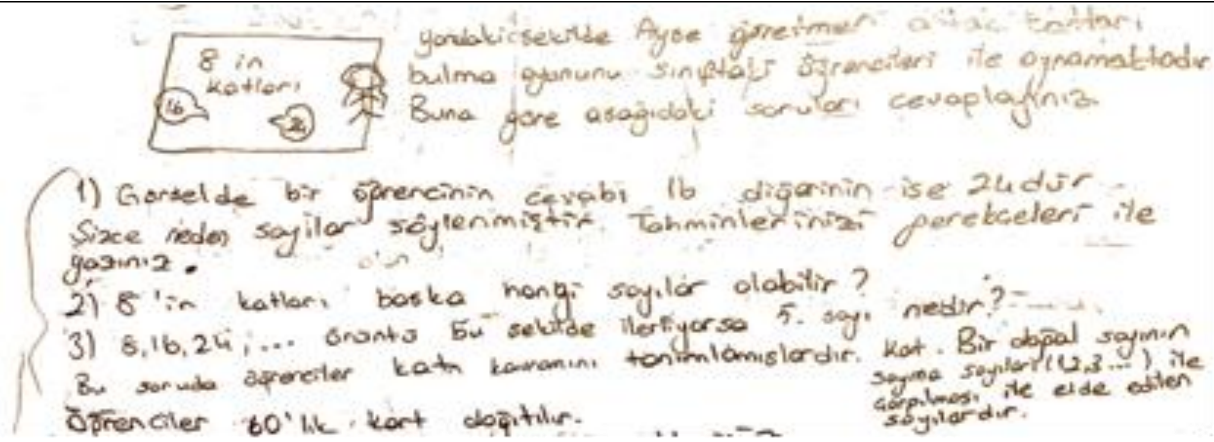
The common multiple problem for PSMT12, which involves dividing a given area into two distinct areas without a remainder, requires that the garden be fully subdivided such that the total area is an exact multiple of both 4 and 6. The constraint that the area must be less than 40 m² necessitates selecting a specific range from the infinite set of common multiples (12, 24, 36, 48, ...).

Some pre-service teachers formulated problem statements using only numbers without any real-life context. Figure 3 presents the problem statement designed by PSMT43, which was evaluated in this context.



"In the figure, Ayşe Teacher is playing a game with her students to identify multiples. Answer the following questions accordingly:

1. In the figure, one student gives the answer 16, while another gives the answer 24. Why do you think these numbers were stated? Explain your reasoning.

2.What other numbers could be multiples of 8?

3.If the pattern 8, 16, 24, ... continues in the same way, what is the fifth term?

Cards numbered from 1 to 60 are distributed to the students."

Figure 3. *The Common Multiple Problem Designed by PSMT43 in the Context of Numbers.*

Problem PSMT43 is designed to help students recognize number relationships by introducing the concept of multiples through a pattern based on multiples of 8. It encourages students to generalize using the numbers 16 and 24 and fosters an understanding that leads to the development of the concept of multiples. The problem primarily focuses on the multiples of a given number.

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The mathematical representations used by the pre-service teachers in solving the common multiple problems they created were examined.

Table 3.

Types and Frequency Distributions of Mathematical Representations Used by Pre-service Teachers When Solving Context-Based Problems Related to Common Multiples

Representation Type	Frequency (f)	Percentage (%)
Table	11	19
Number line	12	20,7
No representation (procedural solution only)	35	60,3

The findings revealed (Table 3) that only 23 pre-service middle school mathematics teachers employed mathematical representations in their solutions. Among these participants, 12 used a number line representation, whereas 11 used a table representation. In contrast, the remaining 35 pre-service teachers did not use any mathematical representation and solved the problems using only procedural calculations.

Representative examples of the mathematical representations identified in the participants' solutions are presented in Figure 4. Figure 4 shows the representation types used by PSMT27 and PSMT37. PSMT27 identified the common multiple by creating a table and matching the relevant values, whereas PSMT37 solved the problem using a number line representation.

Upon examining Figure 4, it is evident that PSMT27 uses a table representation systematically mark the multiples of 18 and 24, enabling students to visually distinguish common multiples (e.g., 72) and obtain organized data. PSMT37, on the other hand, uses a number line representation to show the sequential progression of multiples, demonstrating that the common multiple intersects at the same point. While the table representation fosters awareness primarily through organization, the number line supports understanding of processes and recurring patterns. When these two representations are used together, students’ perspectives are broadened, facilitating a deeper understanding of the concept of common multiples. Therefore, it can be said that the interactive use of these representations plays an effective role in teaching the concept of common multiples.

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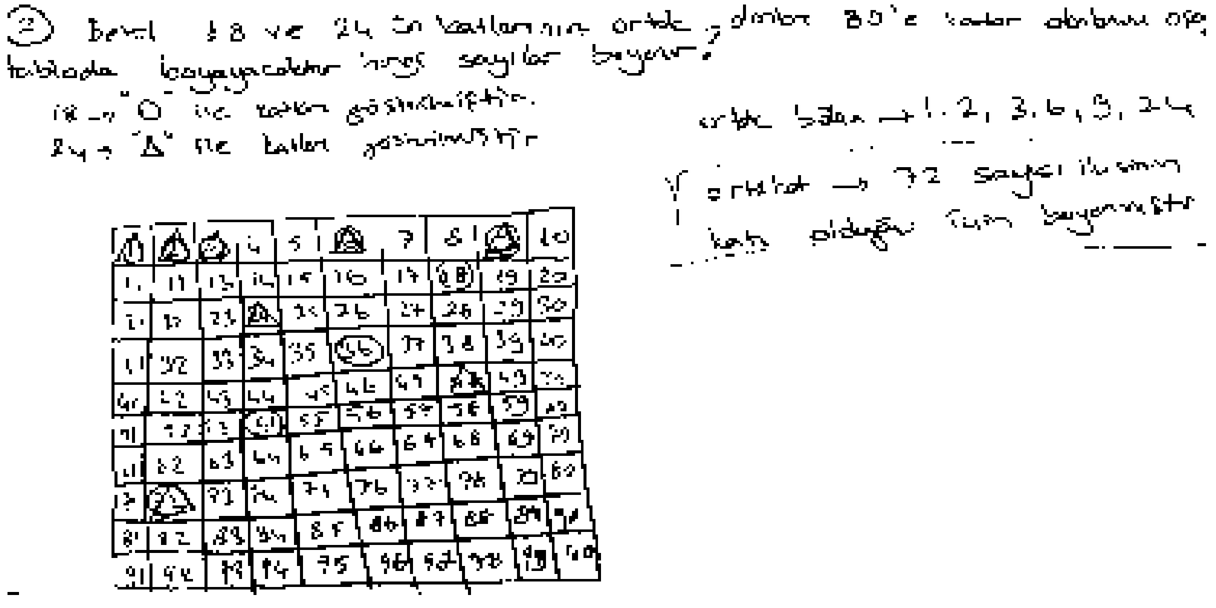
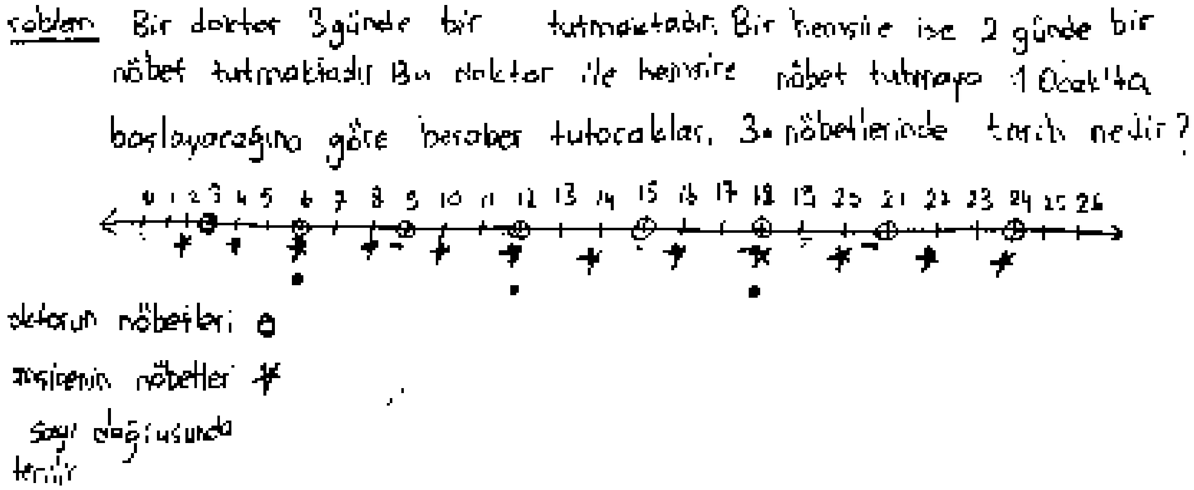


PSMT 27 "Betül is asked to shade the common multiples of 18 and 24 up to 80 in the table. Which numbers should be shaded? The pre-service teacher identified the common multiples by marking the corresponding numbers in the table.



PSMT37:A doctor is on duty every 3 days, while a nurse is on duty every 2 days. If both begin their duty schedules on 1 January, on what date will they be on duty together for the third time? The pre-service teacher used a number line representation to solve the problem by marking the doctor's and the nurse's duty days on the number line.

Figure 4. *Representations Used by PSMT27 and PSMT37 in the Common Multiple Problem.*

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Findings Regarding Common Divisors

The findings regarding the written responses provided by pre-service teachers on the concept of common divisors took into account whether the responses met the relevant learning outcome, whether appropriate contextual information was included within the problem statement, and whether they utilized representational methods while solving the problem. Table 4 first presents the frequencies and percentages, indicating the extent to which the pre-service teachers' written responses met the learning outcome.

Table 4.

Types and Frequency Distributions of Problems Designed by Pre-service Teachers Related to Common Divisors

Problem Types	Frequency (f)	Percentage (%)
Common Divisor Problem	37	63,7
Greatest Common Divisor (GCD) Problem	10	17,2
Problem Unrelated to the Target Concept	5	8,6
No Response	6	10,3

An examination of Table 4 reveals that 37 pre-service teachers designed problems involving the concept of the common divisor. In contrast, 10 pre-service teachers created problems related to the greatest common divisor (GCD), while 5 designed problems that were unrelated to the target concept. In addition, 6 of the 58 pre-service teachers did not provide a response.

While designing problem statements related to the concept of common divisor, the pre-service teachers incorporated various contexts. Table 5 presents the codes corresponding to the themes of the contexts used by the 37 pre-service teachers in their common divisor problems.

Table 5.

Codes and Frequency Distributions Related to the Theme of Contexts Used by Pre-service Teachers in Their Common Divisor Problems

Codes	Frequency (f)	Percentage (%)
Measurement Units (Length / Weight / Price)	33	89,1
Number	4	10,9

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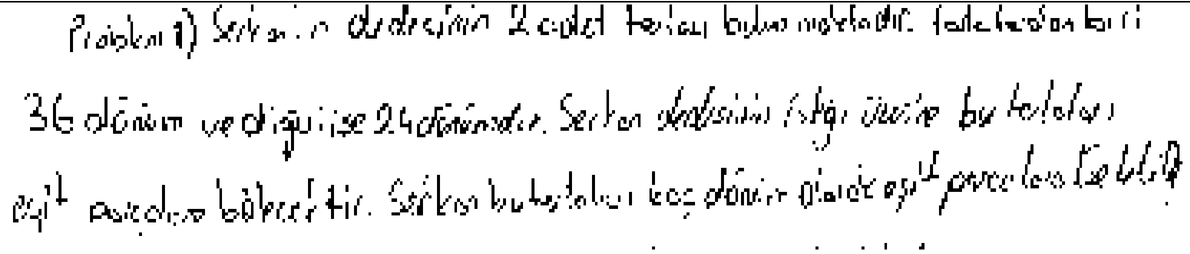
It was determined that the 37 pre-service teachers who were able to create problems involving common divisors provided responses containing different codes within the context of the problem's theme. It was found that 33 pre-service teachers utilized units of measurement when creating problems involving common divisors. In the problems written in this context, the units of measurement used were length, weight, and pricing. Four pre-service teachers, however, created problems that did not include any real-life situations but contained numbers accepted as common divisor problems.

In a large portion of the pre-service teachers' common divisor problems, they included units of measurement. For example, Figure 5 presents the problem statement designed by PSMT12 using the unit of length.


Problem 1) Serkan'ın dedesinin 2 adet tarlası bulunmaktadır. Tarlalardan biri 36 dönüm ve diğeri ise 24 dönümdür. Serkan dedesinin isteği üzerine bu tarlaları eşit parçalara bölecektir. Serkan bu tarlaları kaç dönüm olarak eşit parçalara bölebilir?

Serkan's grandfather has two fields measuring 36 decares and 24 degrees respectively. At his grandfather's request, Serkan will divide both fields into equal-sized plots. What possible plot sizes (in decares) can he use?

Figure 5. *Common Divisor Problem Designed by PSMT12 in the Context of Units of Measurement*

Problem PSMT12 addresses the concept of common divisors in the context of area measurement. The problem presents two fields of different sizes (36 decares and 24 decares) belonging to Serkan's grandfather and asks the student to divide these fields into equal-sized parts. This situation requires students to consider the common divisors of these numbers in order to divide the two different land areas into equal parts. Therefore, the problem involves determining the common divisors of 36 and 24 to establish how many decares each part could be. In this regard, the problem allows students to understand the concept of common divisors not merely as a mathematical operation but within a real-life context of land division and area measurement. Additionally, the "acre" unit used in the problem helps connect the mathematical concept to everyday life. In this context, students may realize that a common unit of

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measurement must be established to divide areas of different sizes into equal parts. This process supports students' abilities to divide into equal parts, think in terms of units of measurement, and interpret the concept of common divisors through a concrete situation. Therefore, the problem can be considered a suitable example that supports the conceptual understanding of the common divisor concept within the context of area measurement.

Figure 6 presents the problem designed by PSMT48 as an example of the problems pre-service teachers designed in the context of numbers involving the concept of common divisor.

Öğretmen kenarları 1br olan eş karelerden 6 tane alacak
şekilde iki gruba verir ve bu karelerden dikdörtgen
yapmalarını ister. 1. grup □□□□□□ şeklinde
ikinci grup □□□ □□□ şeklinde yapar. Öğretmen 1. grubun
kenar sayılarını söylemelerini ister ö 1'e 6
2. grubun ise ö 2'ye 3. Öğretmen 6 sayısının bölenlerinin
1, 2, 3, 6 olduğunu anlatmış olur. Daha sonra öğretmen
bu iki gruba 12 eş birim karelerden verir.
1. grup kenarları 1'e 12, 2'ye 6 oluşturur
2. grup kenarları 3'e 4 olacak dikdörtgen oluşturur.
Buradan da 12'nin bölenleri ö 1, 2, 3, 4, 6, 12 olarak anlatılır.
Öğretmen son olarak 6 ile 12'nin ortak bölenlerinin
1, 2, 3, 6 olduğunu söyler. Öğrenciler ortak bölen kavramını
öğrenirler.

"*The teacher divides six unit squares into two groups and asks each group to construct a rectangle using all six squares. The first group forms a 1 × 6 rectangle, while the second group forms a 2 × 3 rectangle. The teacher then asks each group to state the side lengths of their rectangles. The first group responds 1 and 6, whereas the second group responds 2 and 3. Based on these responses, the teacher explains that the factors of 6 are 1, 2, 3, and 6.*

Next, the teacher gives the two groups twelve unit squares. The first group constructs 1 × 12 and 2 × 6 rectangles, while the second group constructs a 3 × 4 rectangle. The teacher then explains that the factors of 12 are 1, 2, 3, 4, 6, and 12. Finally, the teacher points out that the common divisor of 6 and 12 are 1, 2, 3, and 6, thereby helping the students understand the concept of common divisor."

Figure 6. *Common Divisor Problem Designed by PSMT48 in the Context of Numbers.*

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In the problem designed by PSMT48, the concept of common divisors is introduced through an activity involving the creation of rectangles using unit squares. By having students create different rectangles using 6 unit squares, they can discover the divisors of a number through the lengths of the sides. This approach allows students to connect numbers with geometric modeling. In the final stage, comparing the common divisors of the numbers 6 and 12 helps students develop an understanding of the concept of common divisors.

The mathematical representations used by the pre-service teachers in solving the common divisor problems they created were examined.

Table 6.

Types and Frequency Distributions of Mathematical Representations Used by Pre-service Teachers When Solving Context-Based Problems Related to Common Divisors

Representation Type	Frequency (f)	Percentage (%)
Factor algorithm	7	12,1
Rainbow model	10	17,2
Fraction Strips	7	12,1
No representation (procedural solution only)	34	58,6

The mathematical representations used by the pre-service teachers in solving the common divisor problems they created were examined. The findings revealed that only 24 pre-service middle school mathematics teachers employed mathematical representations in their solutions. Among these participants, 10 used the rainbow model, while 7 used the factor algorithm and another 7 used fraction strips. In contrast, the remaining 34 pre-service teachers did not use any mathematical representation and solved the problems using only procedural calculations.

Representative examples of the mathematical representations identified in the participants' solutions are presented in Figure 7. Figure 7 illustrates the solution produced by PSMT16, who employed both the rainbow model and the factor algorithm to determine the common divisors. The factorization algorithm was used to express the prime factorizations of the numbers systematically, whereas the rainbow model was used to visualize their divisor pairs and identify the common divisors.

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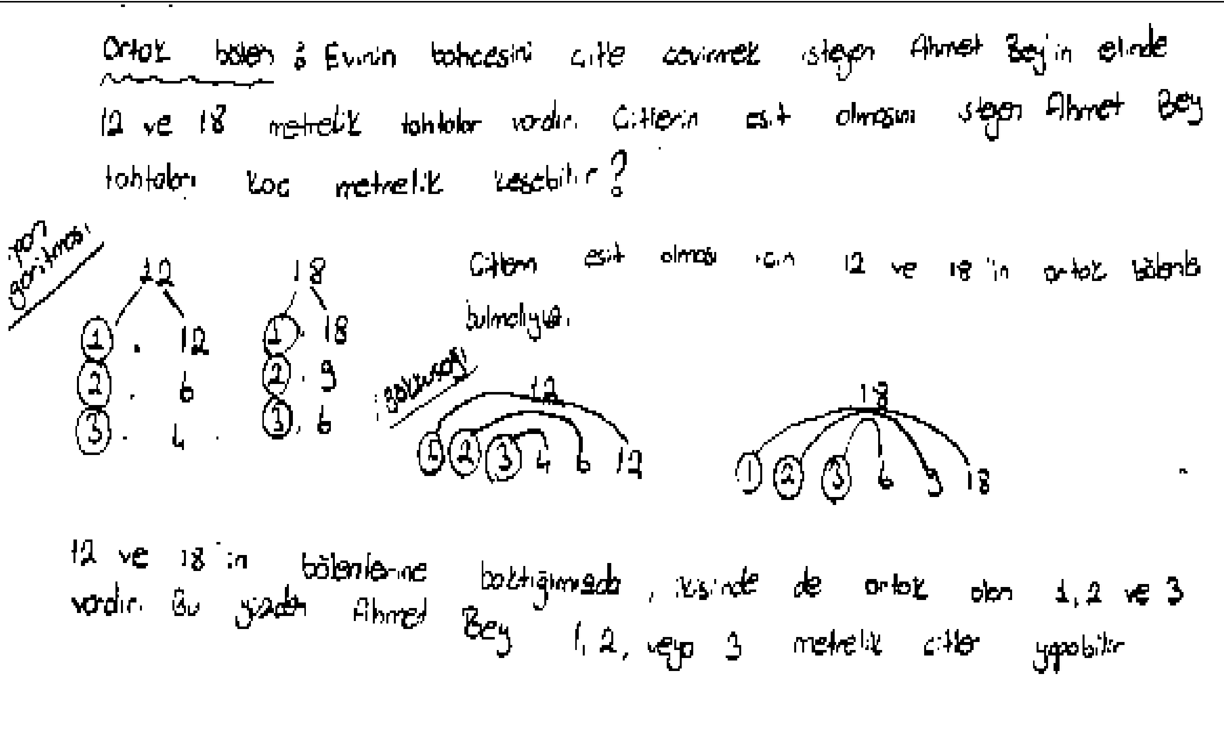

"Mr. Ahmet wants to build a fence around his garden using wooden boards that are 12 meters and 18 meters long. If all fence sections are to be of equal length, into what lengths can the boards be cut? The pre-service teacher used the factor algorithm on the left and the rainbow model on the right."

Figure 7. *Representations Used by PSMT16 in the Common Divisor Problem.*

Figure 7 presents an example of two mathematical representation types: the Factorization Algorithm and the Rainbow Representation. PSMT16 used both representations together in the problem designed to teach the concept of common divisors. The factorization algorithm provides a systematic procedure for expressing the prime factorization of the numbers 12 and 18. In contrast, the rainbow representation visualizes the divisor pairs of each number, enabling students to recognize the multiplicative relationships among the divisors. Using these two representations together allows students to examine the concept of common divisors from different perspectives and supports conceptual understanding through multiple representations. Figure 8 shows the representations used by PSMT50 while solving the problem. PSMT50 explained the solution to the problem using two different representations.

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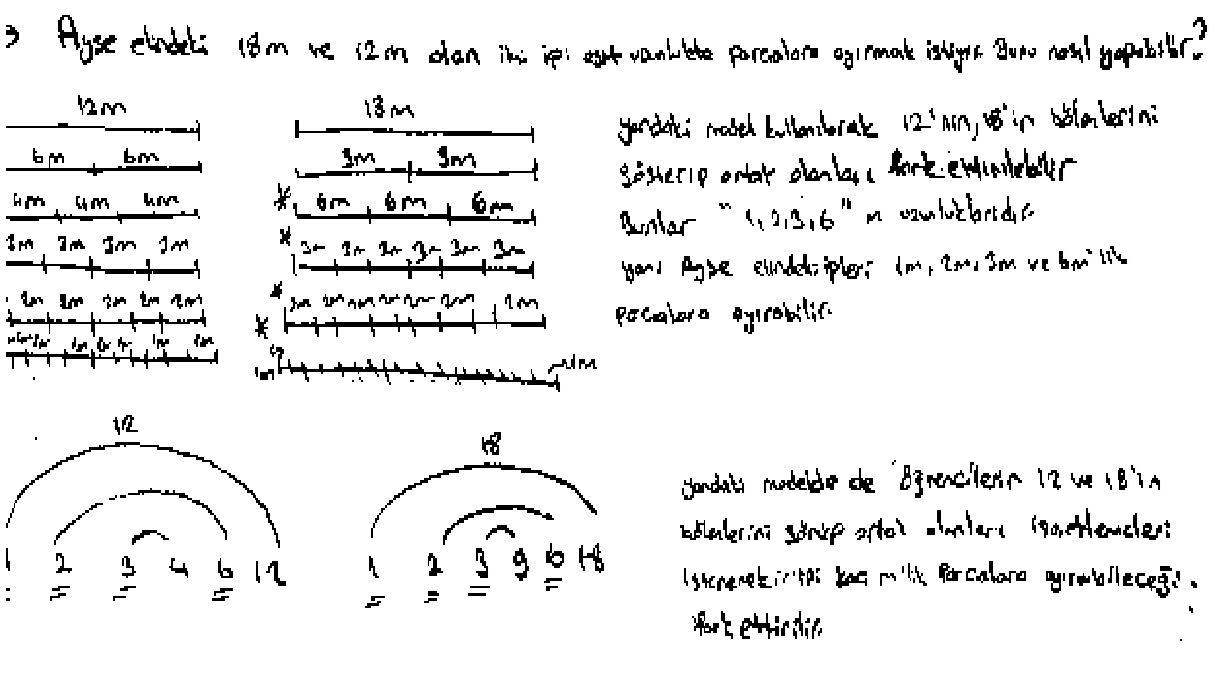

"Ayşe wants to divide two ropes measuring 18 meters and 12 meters into equal-length pieces. The solution is represented using the strip model at the top and the rainbow model at the bottom."

Figure 8. *Representations Used by PSMT50 in the Common Divisor Problem.*

An examination of Figure 8 reveals that the problem designed to teach the concept of common divisors utilizes both the strip model and the rainbow representation. In particular, the model representation which illustrates the process of dividing lengths into segments is expected to help students make sense of the concept in a real-life context by concretizing the division of 12-meter and 18-meter ropes into equal parts. The rainbow representation also illustrates common divisors by systematizing and visualizing the divisors of two numbers. The combined use of these two representations allows students to establish a connection between concrete models and abstract structures.

The written responses of the pre-service teachers were analyzed in accordance with the MAT.6.1.4 learning outcome. During the analysis process, attention was paid not only to whether the participants reached the correct answer but also to whether the problem situations they constructed or selected reflected the intended learning outcome, whether the problem required a common multiple or a common divisor, and the appropriateness of the selected concept to the problem context. Additionally, by identifying the types of representations used by the pre-service teachers, their thinking processes regarding the context-based interpretation of the concepts of common multiples and common divisors were examined from a holistic perspective.

As a result, 41 out of 58 pre-service teachers were able to appropriately design problems involving common multiples and 37 were able to design problems involving common divisor using real-life contexts. However, it was determined that some pre-service teachers did not take into account the fact that the LCM and GCD were not included in the TCEM and created problems that included these concepts. When evaluating the pre-service teachers' use of representations while solving problems, it was determined that they utilized tables, number lines, the factorization algorithm, and visual models. However, most pre-service teachers preferred to generate solutions using a single type of representation, and the use of multiple representations remained limited.

DISCUSSION AND CONCLUSION

This study examined the context-based problems designed by pre-service teachers regarding the concepts of common multiples and common divisors, together with the mathematical representations they used while solving these problems. The findings showed that most participants were able to construct problems within appropriate real-life contexts. However, some participants designed problems involving the concepts of least common multiple (LCM) and greatest common divisor (GCD) rather than common multiples and common divisors. Although the Turkish Century Education Model (MoNE, 2024) emphasizes the concepts of common multiples and common divisors rather than procedural LCM and GCD calculations at this grade level, some pre-service teachers continued to associate these concepts with the procedural knowledge acquired during their previous school experiences. This finding may indicate that conceptual distinctions between common multiples, common divisors, LCM, and GCD have not yet been fully established.

The finding that many pre-service teachers were able to create problem situations related to common multiples and common divisors by using meaningful real-life contexts, particularly time and measurement, suggests that they were able to relate these mathematical concepts to familiar daily situations. Such contexts naturally involve repeated events or equal partitioning situations, making them suitable for developing conceptual understanding of divisibility relationships. This finding is consistent with previous studies reporting that context-based problems facilitate students' understanding of mathematical concepts and support meaningful problem solving (Albayrak et al., 2006; Dede & Yaman, 2005; Zazkis & Campbell, 1996).

Many of the context-based problems designed by the participants reflected characteristics described by Benckert (1997), such as being embedded in meaningful real-life situations, presenting a logical scenario, and requiring a limited number of solution steps. Rather than explicitly directing students to apply a specific procedure, many problems encouraged students to determine the appropriate mathematical concept through the given context. Therefore, the findings of the present study are consistent with Benckert's (1997) emphasis on the role of meaningful contexts in supporting conceptual understanding.

The present study also examined the mathematical representations used by pre-service teachers while solving the problems they had created. Although a number of participants used representations such as tables, number lines, factor algorithms, rainbow models, and fraction strips, the majority relied solely on procedural calculations without using any representation. Similar findings have been reported in previous studies emphasizing that learners frequently rely on procedural approaches when solving mathematical problems (Lithner, 2008). Likewise, Akın and İlhan (2020) highlighted the importance of meaningful contexts in supporting the teaching of common multiples and common divisors.

The limited use of mathematical representations observed in this study suggests that many pre-service teachers continue to approach these concepts procedurally rather than conceptually. Ainsworth (2006) argues that multiple representations play an important role in promoting conceptual understanding by enabling learners to examine mathematical relationships from different perspectives. Consistent with this view, the findings indicate that only a limited proportion of the participants employed mathematical representations while solving their problems, whereas most preferred procedural solution strategies.

When the findings are considered in relation to the emphasis on interpretation in MoNE (2024), it can be argued that pre-service teachers were generally successful in constructing context-based problems; however, greater attention should be given to encouraging the use of mathematical representations and evaluating the mathematical appropriateness of the problems they construct. Learning environments that encourage students to discuss the mathematical structure of the problems they create and to interpret the relationships between problem contexts and mathematical concepts may contribute to the interpretation skills emphasized in the curriculum.

In conclusion, this study examined the context-based problems designed by pre-service middle school mathematics teachers and the mathematical representations they employed while solving these problems. The findings provide insights into how pre-service teachers relate the concepts of common multiples and common divisors to real-life contexts and how they use mathematical representations during problem solving. These findings may contribute to the improvement of mathematics teacher education by emphasizing the importance of conceptual understanding, meaningful contexts, and multiple representations in the teaching of these concepts. Based on these findings, several implications and recommendations for mathematics education and future research are presented below.

Based on the findings of the study, the following recommendations are offered for mathematics educators and future research:

1. Instructional activities that emphasize the interpretation of common multiples and common divisors within meaningful real-life contexts should be incorporated into classroom practice.
2. Learning environments should encourage students to examine the mathematical structure of context-based problems and discuss the relationship between the problem context and the underlying mathematical concepts.
3. Problem situations involving different mathematical structures should be presented comparatively so that students can distinguish between the concepts of common multiples, common divisors, least common multiples, and greatest common divisors.
4. Instructional activities promoting the use of multiple mathematical representations, such as tables, number lines, factor algorithms, rainbow models, and fraction strips, should be integrated into teaching. Combining different representations may strengthen students' conceptual understanding of common multiples and common divisors.
5. Collaborative learning environments should be designed to encourage pre-service teachers to evaluate, compare, and discuss the context-based problems created by their peers, thereby supporting deeper conceptual understanding and reflective thinking.

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Ethical Approval

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Conflict of Interest

None declared